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# Joule-Lenz Law with Hyperbolic Heat Conduction Equation for an Amorphous Chalcogenide Semiconductor

**N.V. Sovtus, K.D. Mynbaev** *

Ioffe Institute, 26 Polytechnicheskaya, Saint-Petersburg, 194021, Russia

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**Abstract**

A heat conduction equation for a chalcogenide glassy semiconductor (CGS) is derived with an account for non-linear temperature dependence of the heat capacity. A possible effect of the dependence on current behavior in CGS is considered. An equation for hypothetical temperature oscillations in CGS is obtained for the case when the system is considered as a medium governed by the Cattaneo-Vernotte-Lykov law for heat flux, which allowed for using hyperbolic heat conduction equation. Formulae for the time dependence of entropy and heat generation in the current filament in CGS are derived. It is shown that current oscillations caused by non-linear temperature dependence of the heat capacity are only possible under extremely strong heat removal with relaxation time exceeding the realistic values by at least three orders of magnitude.

*Keywords*: Joule-Lenz law; Chalcogenide glassy semiconductor; Heat capacity; Current oscillations

## 1. INTRODUCTION

Chalcogenide glassy semiconductors (CGS) represent a broad class of materials incorporating elements of group 16 of the Periodic System (oxygen, sulfur, and tellurium) and possessing a number of unique properties. In these compounds, discovered in the 1950s at Ioffe Institute by B.T. Kolomiets and N.A. Goryunova, the switching effect is observed [1–5]. This is a phenomenon of the reversible breakdown, where the semiconductor can switch millions of times from a high-resistance to a low-resistance state and back without losing its structure. Another important effect typical of CGSs is the memory effect, when the system can remain in a state with a fixed resistance for an unlimited time. These characteristics make CGSs, such as $Ge_2Sb_5Te$ (GST), highly promising in electronic memory devices [1–6] and optical applications, including ultra-fast switching and modulation [6–9].

Switching in CGS is accompanied by current filamentation, a phenomenon, when the current density greatly increases in a specific location [10]. Sometimes, this phenomenon is accompanied by current oscillations with varying amplitude [11].

For several decades, the switching effect has been mostly described using the heat conduction equation. Within the framework of this approach, thermal capacity in the filament (considered as a crystalline cylinder) depends on the temperature and reaches a constant value in a short time [12]. In this work, we explore the possibility of the appearance of current oscillations due to the non-linearly temperature-dependent heat capacity with deriving a corresponding Joule-Lenz law. For this purpose, the differential heat conduction equation (HCE) is solved with an account for temperature-dependent heat capacity, and a general formula for non-linear heat capacity accounting for a non-stationary distribution function is obtained. It is shown that non-linear heat capacity cannot be responsible for temperature (and, therefore, current) oscillations. However, the latter could hypothetically appear under the introduction of a parameter such as relax-

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* Corresponding author: K.D. Mynbaev, e-mail: mynkad@mail.ioffe.ru

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ation time, which appears in the hyperbolic heat conduction equation. This type of equation for describing heat propagation is introduced for accounting the phenomenon of propagation inertia, when it is necessary to consider the finite speed of thermal waves. Still, under this approach the oscillatory process would appear only under extremely strong heat removal. An important result of the work is obtaining the time dependencies for entropy and heat generation in the crystalline filament in CGS.

## 2. SOLUTION WITH NONLINEAR THERMAL CONDUCTIVITY

Current oscillations in CGS as a physical object can be explained either by those of temperature, as it was suggested that the current in a filament is proportional to the square of the maximum temperature [13], or by changes in the volume of the filament. The linear size of the filament forming in CGS when a sample with an area of about 0.1 mm$^2$ is connected to a point contact, is of the order of just few micrometers [11]. Therefore, we suggest representing the filament as a cylinder filled with a gas of phonons with heating uniform along the coordinate (the crystalline local region should contains phonons, which can be considered as a gas). In semiconductors, the term of the HCE responsible for the Joule heating is proportional to $\exp(-\Delta E/k_BT)$, where $\Delta E$ is the activation energy, $k_B$ is Boltzmann's constant, $T$ is the absolute temperature. In CGSs, heating additionally depends on electron tunneling from U-centers, which are lattice defects acting as a “bridge” for the electron's transition to the conduction band. Also, according to Anderson's suggestion [14], electrons additionally undergo interaction with lattice vibrations. This results in the formation of charge carrier pairs resembling the Cooper pairs in a superconductor. Therefore, the Joule heating during the switching effect has a partially quantum nature and the complete HCE can be written accounting for the quantum term given in Ref. [2] as:

$$\rho c\frac{dT}{dt}=\sigma_0F^2\exp\left(\frac{-\Delta E}{k_BT}+\frac{e^2F^2}{3\hbar m}\left(\tau_1+\frac{\hbar}{2k_BT}\right)^3\right)-\lambda\left(T-T_0\right), \tag{1}$$

where $\rho$ is the material density, $t$ is time, $c$ is the heat capacity, $\sigma_0$ is the initial conductivity, $F$ is the electric field, $e$ is the electron charge, $\hbar$ is Planck's constant, $m$ is the effective electron mass, $\tau_1$ is tunneling time, $\lambda$ is the coefficient of heat removal, $T_0$ is the initial temperature.

Let us suggest that the heat capacity non-linearly depends on $T$ within a certain time range, until it reaches a specific constant value. The approximate analytical form for the specific heat capacity was given in Ref. [12] as:

$$c=\frac{3Nk}{m}\left[1-\exp\left(-a\left(T-T_0\right)\right)\right], \tag{2}$$

where

$$a=\frac{u_d\Delta E}{kT_m^2\tau_0}.$$

Here $N$ is the number of unit cells in the crystal, $u_d$ is a scaling parameter, $T_m$ is the maximum temperature (heating limit), and $\tau_0$ is time of relaxation of the phonon gas to the equilibrium state. The latter parameter was calculated using a non-stationary distribution function $f$ obtained from the stationary function $f_0$ by solving the differential equation:

$$\frac{\partial f}{\partial t}=\frac{-f-f_0}{\tau_0}.$$

The heat capacity $c$ is calculated using the distribution function $f$:

$$c=\frac{1}{M}\frac{\partial}{\partial T}\int_0^{\omega_d}\hbar\omega f\left(\omega\right)g\left(\omega\right)d\omega, \tag{3}$$

where $M$ is the crystal mass, $\omega$ is the oscillation frequency, and the expression for the density of states:

$$g\left(\omega\right)=\frac{9N\omega^2}{\omega_D^3}.$$

Here $\omega_D$ is the Debye frequency. As the particles constituting the considered gas are phonons, the function $f$ can be taken as the Bose-Einstein distribution:

$$f_0=\frac{1}{\exp\left(\frac{\hbar\omega}{k_BT}\right)-1}\approx\frac{k_BT}{\hbar\omega}.$$

This expansion is justified simply because the temperatures in the heated region significantly exceed the Debye temperature $T_D$ for the elements constituting common CGSs. For example, for Ge $T_D$ = 360 °C. Considering the above facts, the Boltzmann equation for the phonon gas now appears as:

$$\frac{df}{dt}=\frac{-f}{\tau_0}+\frac{k_BT}{\hbar\omega\tau_0}. \tag{4}$$

To solve the equation above, it is necessary to know the exact time dependence of the absolute temperature. In general form, the solution of the Boltzmann equation is written as:

$$f\left(t\right)=\frac{k_B}{\hbar\omega\tau_0}\exp\left(-\frac{t}{\tau_0}\right)\int_0^t T\left(x\right)dx+\frac{k_BT_0}{\hbar\omega}\exp\left(-\frac{t}{\tau_0}\right). \tag{5}$$

Then, considering the definition of heat capacity and the fact that in Eq. (3) the first derivative with respect to temperature is taken, we see that by differentiating the integral with respect to $T$ we reduce the temperature and transition to a first-order differential equation in time for the temperature, independent of the distribution function's form. A first-order differential equation, with few exceptions, does not describe an oscillatory process. Thus, it can be concluded that there will be no periodic temperature changes in the filament, regardless of the form of the Boltzmann distribution function with the aforementioned right-hand side. Figure 1 shows the Boltzmann kinetic function for a film with thickness $L = 10^{-6}$ m; the temperature dependence on time is considered in more detail in Section 4. The graph of the distribution function can be interpreted as an increase in the probability of a phonon occupying the same energy level with increasing temperature and the transition of the sample to the crystalline phase. At the initial moment of time, the distribution function is equal to the classical Boltzmann function with the assumption that the Boltzmann function describes phonons in an amorphous semiconductor (scales larger than 1 μm are too large). A smooth continuous growth may indicate that the lower time limit for the sample's transition to a crystalline state is $10^{-10}$ s.

## 3. POSSIBLE TEMPERATURE EXTREMES AS TEMPERATURE OSCILLATIONS

We established that regardless of the specific heat capacity formula, it cannot contribute to the oscillations. However, during oscillations of the temperature, at certain time points its time derivative will equal zero, as the temperature reaches extreme values.

Accordingly, it is possible to consider a situation where the right-hand side of Eq. (1) equals zero, and examine the conditions when this could be observed in reality. When $T \to \infty$, the exponent's argument tends to zero, and the exponent can be expanded in a Taylor series. The condition

$$\exp\left(-\frac{\Delta E}{k_B T}\right) \approx 1 - \frac{\Delta E}{k_B T}$$

holds when $T > 1500$ K, considering that $\Delta E = 0.5$ eV. However, with an additional quantum term

$$\frac{e^2 F^2}{3\hbar m}\left(\tau_1 + \frac{\hbar}{2k_B T}\right)^3$$

the temperature, at which the Taylor series expansion is possible, drops to hundreds of Kelvins. Considering this fact, the right-hand side can be expanded as:

$$\sigma_0 F^2\left[1 - \frac{\Delta E}{k_B T} + \frac{e^2F^2}{3\hbar m}\left(\tau_1^3 + \frac{3\hbar\tau_1^2}{2k_B T} + \frac{3\tau_1\hbar^2}{4k_B^2 T^2} + \frac{\hbar^3}{8k_B^3 T^3}\right)\right]$$
$$-\lambda\left(T - T_0\right) = 0.$$

Accordingly, a 4th-degree equation is obtained:

$$\sigma_0 F^2\left[T^3 - \frac{\Delta E T^2}{k_B} + \frac{e^2F^2}{3\hbar m}\left(\tau_1^3 T^3 + \frac{3\hbar\tau_1^2 T^2}{2k_B} + \frac{3\tau_1\hbar^2 T}{4k_B^2} + \frac{\hbar^3}{8k_B^3}\right)\right]$$

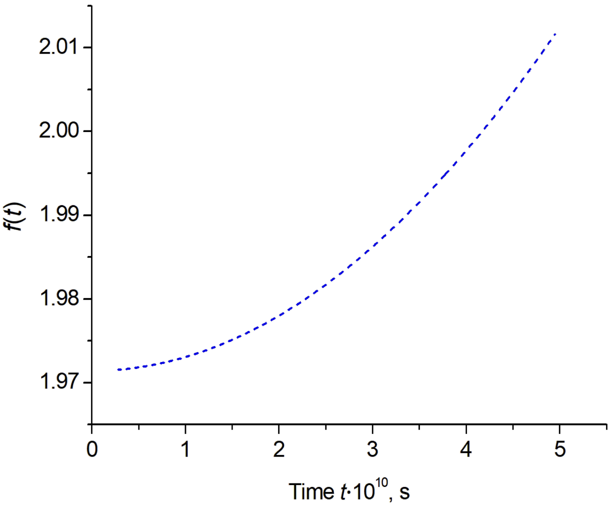

**Fig. 1.** Boltzmann kinetic function for a CGS film with thickness $L = 10^{-6}$ m.

$$-\lambda\left(T^4 - T_0 T^3\right) = 0 \qquad (6)$$

Considering the above, we arrive at the equation (the trivial zero solution is not considered):

$$\left(\frac{e^2F^2\tau_1^3}{3\hbar m} - \lambda\right)T^3 + \left(\lambda T_0 + \frac{e^2F^2\tau_1^2}{2k_B\hbar m} - \frac{\Delta E}{k_B}\right)T^2 + \frac{e^2F^2\tau_1\hbar}{4k_B^2 m}T$$
$$+\frac{e^2\hbar^2F^2}{24k_B^3 m} = 0.$$

This equation must possess real roots, which are also points of temperature extrema. For an electric field tending to infinity:

$$\frac{e^2\sigma_0 F^4\tau_1^3}{3\hbar m} \gg \lambda T_0.$$

Considering that both the heat removal and the initial temperature are low, at electric field $F >> 1$ V/m, and dividing by the factor preceding the cubic temperature term, we obtain:

$$T^3 + \frac{3}{2\tau_1 k_B}T^2 + \frac{3\hbar^2}{4\tau_1^2 k_B^2}T + \frac{3\hbar^3}{24\tau_1^3 k_B^3} = 0. \qquad (7)$$

The dependence on Planck's constant and time indicates that quantum effects are responsible for the emergence of zeros in the temperature derivative; from a classical viewpoint, we obtain a trivial equation with no oscillations. The most interesting case is the one with three real roots for the polynomial (7). Eq. (7) was checked using the Matlab software package; it is easy to see that it has no real positive roots at realistic physical temperatures for a wide range of $F$, $\sigma_0$, and $\lambda$ parameters. Therefore, the presence of extrema is not anticipated either. The derivative of Eq. (7) with respect to $T$ is non-negative for all physically meaningful temperatures, so from the standpoint of an oscillatory process, Eq. (7) is of no interest.

However, the scope of the HCE allows for consideration of another case of heat propagation, namely, the case of the hyperbolic non-stationary equation.

## 4. HYPERBOLIC HEAT CONDUCTION EQUATION

We have found that with the HCE of the first order, the temperature oscillations in the system seem impossible. However, the classical HCE has several limitations. As is known, it is derived from Fourier's law for heat flux $q$:

$$q = -\kappa \nabla T, \tag{8}$$

with κ being the heat exchange coefficient. The authors of Ref. [15] pointed out that Eq. (8) is meaningless at the body-environment boundary when setting a boundary value problem of the first kind (Dirichlet). This is caused by the fact that if the temperature at the body boundary is $T_s$, and the environment temperature is $T_{env}$, the heat flux at the boundary becomes infinite as the coordinate difference tends to zero. To resolve this contradiction, it was proposed to solve the HCE only for a closed system, disregarding the external temperature, or to set boundary conditions of the third kind (Newton's law).

A HCE including the second derivative of temperature with respect to time was proposed in 1958 independently by Vernotte and Cattaneo [16], and was analyzed by Luikov (Lykov) [17]. The result was the Maxwell-Cattaneo equation:

$$q = -\kappa \frac{\partial T}{\partial x} - \tau_r q \frac{\partial T}{dt}, \tag{9}$$

with $\tau_r$ being relaxation (inertia) time. From this equation, it is clear that when a boundary condition of the second kind (Neumann) with zero spatial derivatives is applied at the boundary, the heat flux decays exponentially with the scale of $\tau_r$. It must be added that Eq. (9) was introduced considering a continuous medium; however, heat is transferred through molecular collisions, phonon interactions, etc., and a more accurate approximation should disregard this concept. The second term in Eq. (9) accounts precisely for thermal inertia and the discreteness of the medium. Then the general form of the HCE for current heating in a structure with constants typical of GST will be:

$$\rho c \tau_r \frac{\partial^2 T}{\partial t^2} + \rho c \frac{dT}{dt} = \sigma_0 F^2 \exp\left[\frac{-\Delta E}{k_B T} + \frac{e^2 F^2}{3\hbar m}\left(\tau_1 + \frac{\hbar}{2 k_B T}\right)^3\right]$$
$$-\lambda\left(T - T_0\right) + \kappa \Delta T. \tag{10}$$

However, in the zero approximation, one can consider the body's heating as homogeneous, as we did before. This equation is called the hyperbolic HCE. The inertia time $\tau_r$ is a parameter responsible for the finite propagation speed of heat waves in the material. The speed of the thermal wave can be described by the formula:

$$v = \sqrt{\alpha^2 / \tau_r},$$

where $\alpha^2$ is the thermal diffusivity. In the classical HCE under Fourier's law, the speed of heat propagation is infinite. The hyperbolic HCE is applicable for small system sizes (comparable to the parameter $v \cdot \tau_r$). The subsequent calculation is theoretical, as for GST the relaxation time τ is known very approximately. This parameter also cannot be considered as a rigidly fixed constant; in general, the relaxation time depends on temperature and in dielectrics at cryogenic temperatures is ~ $10^{-11}$ s with thermal wave propagation speed being ~ $10^3$ m/s.

The cubic term with inverse temperature is not the only possible additional term in the exponential factor. To develop a general model, the equation should be written as:

$$\rho c \tau_r \frac{\partial^2 T}{\partial t^2} + \rho c \frac{dT}{dt} = \sigma_0 F^2 \exp\left(-\frac{\Delta E}{k_B T} + \frac{A}{T^\gamma}\right) - \lambda\left(T - T_0\right), \tag{11}$$

where $A$ and γ are constants.

Without the additional term, approximating the exponent with a linear term in temperature is only possible in a narrow temperature range (≤ 20 K) near $T_0$. The second term has dependence on the electric field $F$, and in its presence, the exponential term can be approximated by a linear function over a wider range. Let us expand the exponential heating term in a Taylor series; then the HCE will take the following form:

$$\frac{d^2\theta}{dt^2} + \frac{1}{\tau_r}\frac{d\theta}{dt} + \omega_0^2 \theta = C, \tag{12}$$

where

$$\theta = T - T_0,$$

$$\omega_0^2 = \frac{\lambda - \sigma_0 F^2 f'(T_0)}{\rho c \tau_r},$$

$$C = \frac{\sigma_0 F^2 f\left(T_0\right)}{\rho c \tau_r},$$

$$f\left(T_0\right) = \exp\left(-\frac{\Delta E}{k_B T_0} + \frac{A}{T_0^\gamma}\right),$$

$$f'(T_0) = f\left(T_0\right)\left(\frac{\Delta E}{k_B T_0^2} - \frac{\gamma A}{T_0^{-\gamma+1}}\right),$$

$$\theta\left(t\right) = \exp\left(-\frac{t}{2\tau_r}\right)\left(D_1 \cos \omega t + D_2 \sin \omega t\right) + \frac{C}{\omega_0^2},$$

$$\omega = \sqrt{\omega_0^2 - \frac{1}{4\tau_r^2}},$$

$$\theta\left(0\right) = 0 \frac{d\theta\left(0\right)}{dt} = 0.$$

The explicit $T(t)$ dependence is:

$$T(t) = T_0 + \frac{C}{\omega_0^2} - \exp\left(-\frac{t}{2\tau_r}\right)\left(\frac{C}{\omega_0^2}\cos\omega t + \frac{C}{2\omega\tau_r\omega_0^2}\sin\omega t\right), \tag{13}$$

The period of the obtained oscillations is $\check{T} = 2\pi/\omega$, and the amplitude decays exponentially. Over time, the temperature approaches a final value equal to $T_0 + C/\omega_0^2$. We emphasize the fact that the $T_m$ does not depend on $\tau$.

Under the condition

$$\lambda - \sigma_0 F^2 f'(T_0) \leq \frac{\rho c}{4\tau_r} \tag{14}$$

there will be no temperature oscillations. That is, they appear in the system with low heat removal, high electric field, and short inertia time, leading to an instantaneous system response to heating. In the ordinary first-order HCE, the relaxation time is zero, which leads to an infinitely large imaginary part of $\lambda$, and from the point of view of mathematics, renders the oscillations impossible.

If we let $\tau$ tend to zero, simulating the situation described by the parabolic classical HCE, of interest is the situation where $F$ tends to infinity. Then we can neglect heat removal and find that the temperature tends to a homogeneous solution:

$$T(t) \to T_0 + \frac{1}{\left(-\frac{\Delta E}{k_B T_0^2} + \frac{\gamma A}{T_0^{-\gamma+1}}\right)}.$$

With an infinitely large field and no heat removal, the temperature should also tend to an infinitely large value, which allows us to define the value of parameter $A$ as

$$A = \frac{\Delta E \cdot T_0^{-\gamma-1}}{k_B \cdot \gamma}.$$

Note that if the parameter $A$ depends directly on the field $F$, we obtain an infinite temperature as a solution for an infinitely large field, and the exponential factor on the right-hand side depends only on the activation energy (purely thermal breakdown). However, this is only a hypothesis; it must be added that the dependence of $A$ on the initial temperature implies the presence of a certain thermal "memory," where the semiconductor possesses properties of a thermistor, and the expression for the *I-V* characteristic incorporates the initial temperature $T_0$. In real cases, $A$ carries information about the structure of the semiconductor material, but currently, there is no information on *I-V* characteristic of semiconductor films being dependent on the initial temperature. If we consider the HCE for a formula like Eq. (10), accounting for quantum effects, the $T(t)$ dependence will look like:

$$T(t) = T_0 + \frac{\sigma_0 F^2 f(T_0)}{\lambda - \sigma_0 F^2 f'(T_0)}$$

$$\times\left[1 - \exp\left(-\frac{t}{2\tau_r}\right)\left(\cos\omega t + \frac{1}{2\omega\tau_r}\sin\omega t\right)\right], \tag{15}$$

where

$$\omega = \sqrt{\frac{\lambda - \sigma_0 F^2 f'(T_0)}{\rho c \tau_r} - \frac{1}{4\tau_r^2}} f(T_0)$$

$$= \exp\left[-\frac{\Delta E}{k_B T_0} + \frac{e^2 F^2}{3\hbar m}\left(\tau_1 + \frac{\hbar}{2k_B T_0}\right)^3\right],$$

$$f'(T_0) = f(T_0)\left(\frac{\Delta E}{k_B T_0^2} - \frac{e^2 F^2\left(\tau_1 + \frac{\hbar}{2k_B T_0}\right)^2}{2mk_B T_0^2}\right).$$

Analysis of formula (15) shows that oscillations do not occur for realistic physical parameters of GST; the oscillatory process requires colossal heat removal and high relaxation time: $\lambda > 10^8$ W/(m$^3$·K) and $\tau_r > 10^{-5}$ s.

## 5. JOULE-LENZ LAW

The aforementioned hyperbolic HCE was considered homogeneous in coordinate, so the ultimate total heat generated according to the Joule-Lenz law is defined as the time integral:

$$Q = \int \sigma(T) F^2 V dt, \tag{16}$$

where $V$ is the filament volume. It is noteworthy that during temperature oscillations, when the temperature decreases, the entropy of the phonon gas in the filament also decreases. This does not violate the second law of thermodynamics, as the filament is an open system, and Ref. [13] considers the option of lateral heat removal. Let us consider the standard formula for entropy as a time integral, where $r_f$ is the filament radius:

$$S = \int \frac{\sigma(T) F^2 \pi r_f^2 L}{T} dt. \tag{17}$$

For convenience, it is also sufficient to expand the conductivity in a Taylor series.

A more detailed expression for the entropy will look as:

$$S(t) = \sigma_0 F^2 V\left[f'(T_0)\cdot t + \int_0^t \frac{f(T_0) - f'(T_0)T_0}{T(u)} du\right]. \tag{18}$$

Figures 2, 3, and 4 show time dependences of the specific entropy, temperature, and heat generation (for hyperbolic HCE, and taking into account tunneling between adiabatic terms). In calculations, the following parameters were used: $m = 0.1m_0$, $T = 300$ K, $\tau_1 = 10^{-12}$ s, $F = 10^5$ V/m, $\Delta E = 0.1$ eV, $\sigma_0 = 10^3$ (Ohm·m)$^{-1}$, $\lambda = 10^7$ W/m$^3$, $\tau_r = 10^{-9}$ s, $L = 10^{-6}$ m, $\rho = 6000$ kg/m$^3$, $c = 200$ J/(kg·K).

The entropy dependence on time is nearly linear (Fig. 2). In terms of the heating magnitude in Kelvins, the tempera-

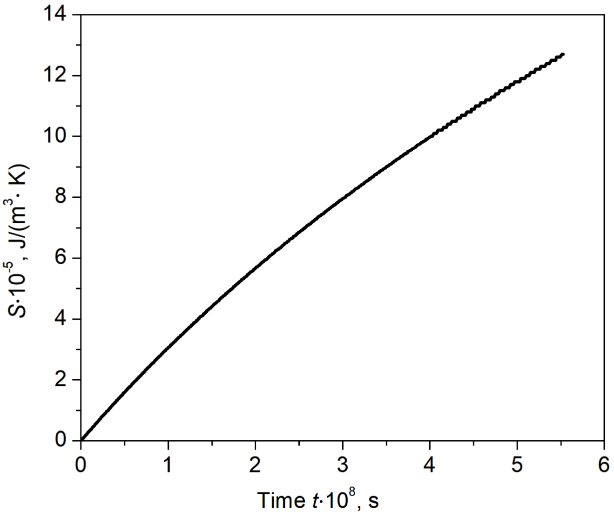

**Fig. 2.** Entropy dependence on time.

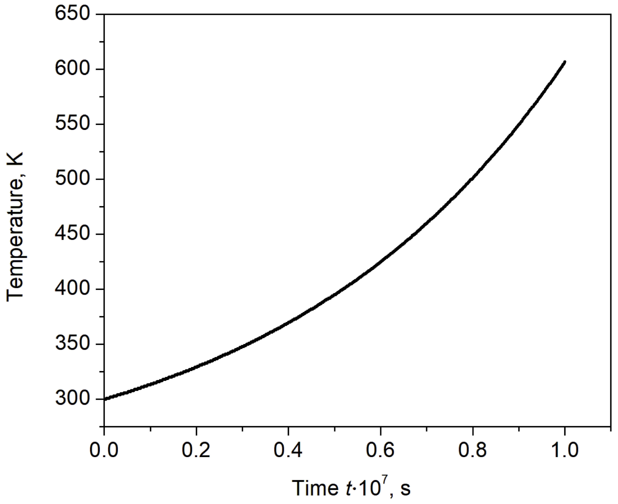

**Fig. 3.** Temperature *vs* time.

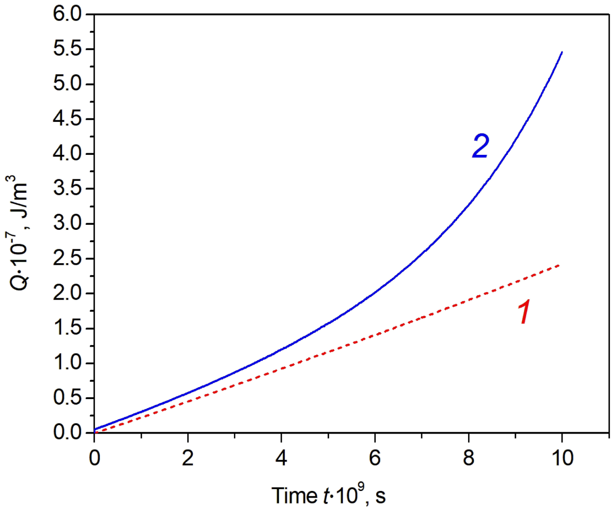

**Fig. 4.** Heat release *vs* time for two $F$ values: $10^5$ V/m (curve *1*) and $5\times10^5$ V/m (curve *2*).

ture calculated for the hyperbolic HCE coincides with the temperature experimentally estimated in Ref. [10]. However, in the case of Eq. (15), heating occurs at voltages several times smaller. One reason for the discrepancy is the extremely small effective electron mass $m$ used in our equation, which creates a “short circuit” effect: the material's low resistance leads to increased heat generation. It is also necessary to consider that the exponential factor in the right-hand side was represented as a first-degree polynomial, which also played a role in the discrepancy. The scale of heat generation coincides with the experimentally obtained $Q \sim 10^8$ J/m$^3$ at $V \sim 10^{-20}$ m$^3$ (with both $L$ and $r_f$ being of the order $10^{-6}$ m). Time required for heating by one hundred Kelvins is approximately 40 ns. This time scale is of the same order as the cooling time scale of the filament during the first microsecond after the switching effect [11].

Figure 4 shows that increasing the field by an order of magnitude cause a rapid growth in heat release over a shorter time.

For systems with impaired heat removal, more complex versions of the thermodynamic equation were proposed, for example, Modified Hyperbolic Equation [18], Modified Internal Energy [19], and the equation with ballistic-diffusive phonon motion [20].

## 6. CONCLUSION

This work established that non-linear temperature dependence of heat capacity cannot be responsible for the occurrence of current oscillations in CGS under switching. A general analytical formula for the Boltzmann kinetic distribution function in the non-equilibrium case was derived. For an electric field tending to infinity, the rate of temperature increase over time is represented by a polynomial of the absolute temperature with coefficients depending on Planck's constant and the tunneling time in the Anderson model. If a second time derivative, accounting for the inertia of heat flux propagation, is introduced into the heat conduction equation, temperature oscillations over time become hypothetically possible; however, this requires colossal heat removal and a relaxation time much larger than the real one. Dependencies for temperature, heat generation, and entropy on time were found for real GST parameters. It was shown that the hyperbolic heat conduction equation describes thermodynamic quantities, such as the amount of heat and entropy obtained in experiments on GST films, with reasonable accuracy.

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# Закон Джоуля-Ленца с гиперболическим уравнением теплопроводности для аморфного халькогенидного полупроводника

**Н.В. Совтус, К.Д. Мынбаев**

Физико-технический институт им. А.Ф. Иоффе, Политехническая ул., д. 26, Санкт-Петербург, 194021, Россия

**Аннотация.** Получено уравнение теплопроводности для халькогенидного стеклообразного полупроводника (ХСП) с учетом нелинейной температурной зависимости теплоемкости. Рассмотрено возможное влияние этой зависимости на поведение тока в ХСП. Получено уравнение для гипотетических температурных колебаний в ХСП в случае, когда система рассматривается как среда, описываемая законом Каттанео-Вернотте-Лыкова для теплового потока, что позволило использовать гиперболическое уравнение теплопроводности. Выведены формулы для временной зависимости энтропии и тепловыделения в шнуре тока в ХСП. Показано, что колебания тока, вызванные нелинейной температурной зависимостью теплоемкости, возможны только при чрезвычайно сильном теплоотводе со временем релаксации, превышающим реалистичные значения как минимум на три порядка.

*Ключевые слова*: закон Джоуля-Ленца; халькогенидный стеклообразный полупроводник; фазовый переход; теплоемкость; колебания тока